Class XII Session 2025-26
Subject - Mathematics
Sample Question Paper - 4

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. Let A = {1, 2, 3} and let R = {(1,1), (2, 2), (3, 3), (1, 2), (2,1), (2, 3), (3, 2)}. Then, R is **[1]**

a) reflexive and symmetric but not transitive
b) an equivalence relation
c) symmetric and transitive but not reflexive
d) reflexive and transitive but not symmetric

2. The value of $\sin^{-1}(\cos\frac{\pi}{9})$ is **[1]**

a) $\frac{5\pi}{9}$
b) $\frac{7\pi}{18}$
c) $\frac{-5\pi}{9}$
d) $\frac{\pi}{9}$

3. For which value of x, are the determinants $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix}$ and $\begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$ equal? **[1]**

a) ± 2
b) 2
c) ± 3
d) -3

4. The function f(x) = |cos x| is **[1]**

a) everywhere continuous but not differentiable at (2n + 1) $\frac{\pi}{2}$, n $\in$ Z
b) either continuous or differentiable at (2n + 1) $\frac{\pi}{2}$, n $\in$ Z
c) neither continuous nor differentiable at (2n + 1) $\frac{\pi}{2}$, n $\in$ Z
d) everywhere continuous and differentiable

5. If a vector makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and y-axis, then the angle which it **[1]**

makes with positive z-axis is:

a) 0
b) $\frac{3\pi}{4}$
c) $\frac{\pi}{2}$
d) $\frac{\pi}{4}$

6. Consider the following statements in respect of the differential equation $\frac{d^2y}{dx^2} + \cos\left(\frac{dy}{dx}\right) = 0$ **[1]**

i. The degree of the differential equation is not defined.

ii. The order of the differential equation is 2.

Which of the above statement(s) is/are correct?

a) Neither (i) nor (ii)
b) Both (i) and (ii)
c) Only (ii)
d) Only (i)

7. The graph of the inequality 2x + 3y > 6 is **[1]**

a) whole XOY – plane excluding the points on the line 2x + 3y = 6.
b) half plane that neither contains the origin nor the points of the line 2x + 3y = 6.
c) entire XOY plane.
d) half plane that contains the origin.

8. Magnitude of the vector $\vec{a} = 2\hat{i} - 7\hat{j} - 3\hat{k}$ is **[1]**

a) $\sqrt{61}$
b) $\sqrt{65}$
c) $\sqrt{62}$
d) $\sqrt{63}$

9. $\int \frac{dx}{(2-3x)} = ?$ **[1]**

a) - 3 log |2 – 3x| + C
b) log |2 + 3x| + C
c) - log |2 – 3x| + C
d) $-\frac{1}{3}\log|2-3x| + C$

10. The order of matrix A $= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & -1 \end{bmatrix}$ is ________. **[1]**

a) 2×3
b) 3×3
c) 2×2
d) 3×2

11. The point at which the maximum value of x + y, subject to the constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x, y $\geq$ 0 is obtained, is **[1]**

a) (30, 25)
b) (20, 35)
c) (35, 20)
d) (40,15)

12. For what value of λ, the projection of vector $\hat{i} + \lambda\hat{j}$ on vector $\hat{i} - \hat{j}$ is $\sqrt{2}$? **[1]**

a) 0
b) -1
c) 1
d) 3

13. $\begin{vmatrix} \cos 70^\circ & \sin 20^\circ \\ \sin 70^\circ & \cos 20^\circ \end{vmatrix}$ = ? **[1]**

a) sin 50^o
b) 0
c) cos 50^o
d) 1

14. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement the the probability that exactly two of the three balls were red, the first ball being red, is **[1]**

a) $\frac{4}{7}$

b) $\frac{5}{28}$

c) $\frac{15}{28}$

d) $\frac{1}{3}$

15. General solution of y log y dx – x dy = 0 **[1]**

a) $y = e^{cx}$

b) $y^2 = e^{cx}$

c) $y = e^{cx} + e^{-cx}$

d) $y = e^{-cx}$

16. Let L denote the set of all straight lines in a plane. Let a relation R be defined by lRm if and only if l is perpendicular to m ∀ l, m ∈ L. Then R is **[1]**

a) reflexive

b) symmetric

c) Asymmetric

d) transitive

17. For what value of k may the function f(x) = $\begin{cases} \mathrm{k}\left(3\mathrm{x}^2-5\mathrm{x}\right), & \mathrm{x} \leq 0 \\ \cos \mathrm{x}, & \mathrm{x}>0 \end{cases}$ become continuous? **[1]**

a) 1

b) $-\frac{1}{2}$

c) 0

d) No value

18. The straight line $\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$ is **[1]**

a) perpendicular to z-axis

b) parallel to x-axis

c) parallel to y-axis

d) parallel to z-axis

19. **Assertion (A):** The rate of change of area of a circle with respect to its radius r when r = 6 cm is 12π cm^2/cm. **[1]**
Reason (R): Rate of change of area of a circle with respect to its radius r is $\frac{dA}{dr}$, where A is the area of the circle.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. The value of λ for which the two vectors $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\hat{k}$ are perpendicular is **[1]**

a) 8

b) 4

c) 6

d) 2

Section B

21. Evaluate: $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$. **[2]**

OR

Write the interval for the principal value of function and draw its graph: $\tan^{-1}$ x.

22. Find all the points of discontinuity of the greatest integer function defined by f(x) = [x], where [x] denotes the greatest integer less than or equal to x. **[2]**

23. Find the intervals in which $f(x)=(x+2)e^{-x}$ is increasing or decreasing. **[2]**

OR

If $x^{30}y^{20}=(x+y)^{50}$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

24. Evaluate: $\int e^x(\cot x - \operatorname{cosec}^2 x)\,dx$ **[2]**

25. If A = $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, find AA^T **[2]**

Section C

26. Using integration, find the area of the region bounded by the lines x - y = 0, 3x - y = 0 and x + y = 12. **[3]**

27. A girl walks 4 km towards west, then she walks 3 km in a direction 30^0 east of north and stops. Determine the girl's displacement from her initial point of departure. **[3]**

28. Evaluate the definite integral $\int_0^\pi \frac{1}{1+\sin x} dx$ **[3]**

OR

Evaluate: $\int \frac{1}{5-4\cos x} dx$

29. Find the general solution of the differential equation: $(x^2 + 1)\frac{dy}{dx} - 2xy = (x^2 + 1)(x^2 + 2)$ **[3]**

OR

Solve the differential equation: $\left[x\sqrt{x^2 + y^2} - y^2\right]$ dx + xy dy = 0

30. Solve the Linear Programming Problem graphically: **[3]**

Maximize Z = x + y Subject to

$-2x + y \leq 1$

$x \leq 2$

$x + y \leq 3$

$x, y \geq 0$

OR

Determine graphically the minimum value of the objective function Z = -50x + 20y subject to the constraints:

$2x - y \geq -5$

$3x + y \geq 3$

$2x - 3y \leq 12$

$x \geq 0, y \geq 0$

31. Show that the function f(x) defined by f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$ is continuous at x = 0. **[3]**

Section D

32. Evaluate: $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx.$ **[5]**

33. Let A = R - {3}, B = R - {1]. If $f : A \rightarrow B$ be defined by $f(x) = \frac{x-2}{x-3}\ \forall x \in A$. Then, show that f is bijective. **[5]**

OR

Let A = {1, 2, 3, ...9} and R be the relation in $A \times A$ defined by (a, b) R (c, d) if a + d = b + c for (a, b), (c, d) in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class [(2, 5)].

34. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find the value of A^{-1}. **[5]**

Using A^{-1}, solve the system of linear equations:

x - 2y = 10,

2x - y - z = 8,

-2y +z = 7

35. Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ intersect and find their point of intersection. **[5]**

OR

Find the length shortest distance between the lines: $\frac{x-3}{3} = \frac{y-8}{-1} = z - 3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Akash and Prakash appeared for first round of an interview for two vacancies. The probability of Nisha's selection is $\frac{1}{3}$ and that of Ayushi's selection is $\frac{1}{2}$.

i. Find the probability that both of them are selected. (1)
ii. The probability that none of them is selected. (1)
iii. Find the probability that only one of them is selected.(2)

OR

Find the probability that atleast one of them is selected. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

Based on the above information, answer the following questions.

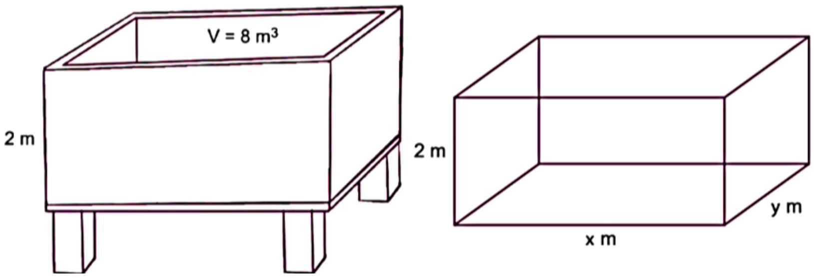
i. If ₹ 15000 is invested in bond X, then what is the matrix representation of A and B? (1)
ii. If ₹ 15,000 is invested in bond X, how can we determine the total amount of interest received on both bonds? (1)
iii. How much is the investment in two bonds if the trust fund obtains an annual total interest of ₹3200? (2)

OR

What is the amount of investment in bond Y if the interest given to the old age home is ₹500? (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



i. Express making cost C in terms of length of rectangle base. (1)

ii. If x and y represent the length and breadth of its rectangular base, then find the relation between the variables. (1)

iii. Find the value of **x** so that the cost of construction is minimum. (2)

OR

Verify by second derivative test that cost is minimum at a critical point. (2)

Solution

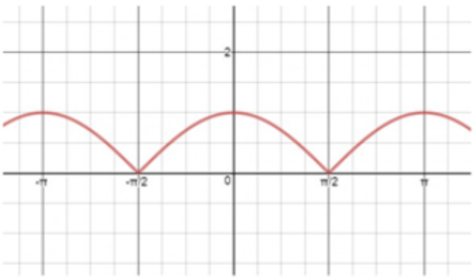
Section A

1. **(a)** reflexive and symmetric but not transitive
   Explanation:
   reflexive and symmetric but not transitive .
   Reflexivity and transitivity follows from definition.
   Here,(3,2) ,(2,1) are in R but (3,1) is not in R,so R is not transitive.

2. **(b)** $\frac{7\pi}{18}$
   Explanation:
   $\sin^{-1}(\cos\frac{\pi}{9}) = \sin^{-1}(\sin\left(\frac{\pi}{2}-\frac{\pi}{9}\right)) = \sin^{-1}(\sin\frac{7\pi}{18}) = \frac{7\pi}{18}$

3. **(a)** ±2
   Explanation:
   ±2
   $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$
   $2x^2 + 15 = 20 + 3$
   $2x^2 = 23-15$
   $2x^2 = 8$
   $x^2 = 4$
   $x = \pm2$

4. **(a)** everywhere continuous but not differentiable at $(2n + 1)\frac{\pi}{2}, n \in Z$
   Explanation:
   Given that $f(x) = |\cos x|$

   From the graph it is evident that it is everywhere continuous but not differentiable at $(2n + 1)\frac{\pi}{2}, n \in Z$

5. **(c)** $\frac{\pi}{2}$
   Explanation:
   $\frac{\pi}{2}$

6. **(b)** Both (i) and (ii)
   Explanation:
   Both (i) and (ii)

7. **(b)** half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

Explanation:

The inequality 2x + 3y > 6 represents a half-plane where points satisfy the condition of being above the line 2x + 3y = 6. The line 2x + 3y = 6 is not included in the solution, since the inequality is strict (">" and not "geq").

To determine which half-plane, we can test a point not on the line. Testing the origin (0, 0) : Substitute x = 0 and y = 0 into the inequality: 2(0) + 3(0) = 0 Since 0 $\not>$ 6, the origin is not in the solution region.

Thus, the solution is a half-plane that does not contain the origin nor the points on the line.

The correct option is: half-plane that neither contains the origin nor the points of the line 2x + 3y = 6.

8.

(c) $\sqrt{62}$

Explanation:

We have:

$\vec{a}=2\hat{i}-7\hat{j}-3\hat{k}$,

then,

$\left|\overrightarrow{a}\right|=\sqrt{2^2+(-7)^2+(-3)^2}=\sqrt{4+49+9}=\sqrt{62}$.

9.

(d) $-\frac{1}{3}\log|2-3x|+C$

Explanation:

Formula :- $\int x^n dx=\frac{x^{n+1}}{n+1}+c; \int \frac{1}{x^2}dx=\log x+c$

Therefore ,

Put 2 - 3 x = t

-3 dx = dt

$=\frac{1}{3}\int\frac{1}{t}dt$

$=-\frac{1}{3}\log t+c$

$=-\frac{1}{3}\log|2-3x|+c$

10. **(a)** 2×3

Explanation:

Order of a matrix is given by

(number of rows) $\times$ (number of columns)

$\therefore$ Order of matrix A = 2 $\times$ 3

11.

(d) (40,15)

Explanation:

We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

x + 2y = 70, 2x + y = 95, x = 0 and y = 0

Region represented by x + 2y $\leq$ 70 :

The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation x + 2y $\leq$ 70 . So, the region containing the origin represents the solution set of the inequation x + 2y $\leq$ 70.

Region represented by 2x + y $\leq$ 95 :

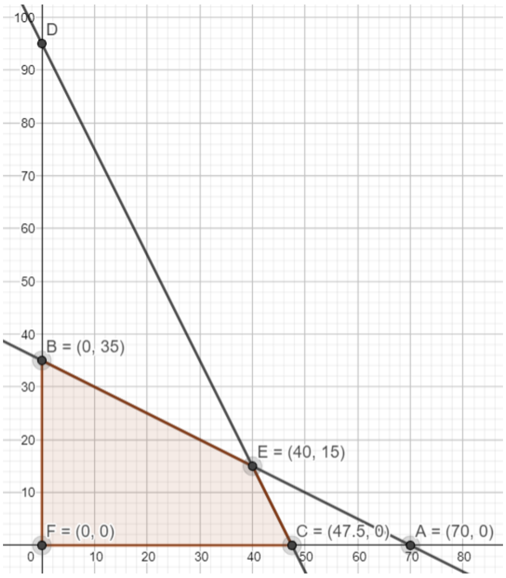
The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2},0\right)$ respectively. By joining these points we obtain the line 2x + y = 95

Clearly (0, 0) satisfies the inequation 2x + y $\leq$ 95 . So, the region containing the origin represents the solution set of the inequation 2x + y $\leq$ 95

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0

The feasible region determined by the system of constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x $\geq$ 0, and y $\geq$ 0 are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2}, 0\right) : \frac{95}{2}+0=\frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

12.

(b) -1

Explanation:

-1

13.

(b) 0

Explanation:

By evaluating given determinant and using sin (90 - A) = cos A. we get value of det. = 0

14. **(a)** $\frac{4}{7}$

Explanation:

Let E_1 = Event that first ball is red = (RRR, RRB, RBR, RBB)

And E_2 = Event that exactly two of three balls being red = (RRR, RRB)

$\therefore P\left(E_1\right)=P_R \cdot P_R \cdot P_R+P_R \cdot P_R \cdot P_B+P_R \cdot P_B \cdot P_R+P_R \cdot P_B \cdot P_B$

$=\frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6}+\frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6}+\frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6}+\frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6}$

$=\frac{60+60+60+30}{336}=\frac{210}{336}$

$P\left(E_1 \cap E_2\right)=P_B \cdot P_R \cdot P_R+P_R \cdot P_R \cdot P_R$

$=\frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6}+\frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6}=\frac{120}{336}$

$\therefore P\left(E_2 / E_1\right)=\frac{P\left(E_1 \cap E_2\right)}{P\left(E_1\right)}=\frac{120 / 336}{210 / 336}=\frac{4}{7}$

15. **(a)** $y=e^{cx}$

Explanation:

$y \log y \, dx=x \, dy$

$\int \frac{1}{x} dx=\int \frac{1}{y \log y} dy$

$\log|x|=\log|\log y|+\log C$ Since $\int \frac{f'(x) dx}{f(x)}=\log|f(x)|+c$ and $\frac{1}{C}=c$ a new constant

$\log x=\log(C \log y)$

$x = C \log y$
$\log y = \frac{1}{C} x$
$\log y = cx$
$y = e^{cx}$

16.

(b) symmetric

Explanation:

Let (x,y)∈R , such that x⊥y.

We can also write from above that, y⊥x.

Hence, (y,x)∈R

So, They are symmetric.

17.

(d) No value

Explanation:

No value

18. **(a)** perpendicular to z-axis

Explanation:

We have,

$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

Also, the given line is parallel to the vector $\vec{b} = 3\hat{i} + \hat{j} + 0\hat{k}$

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

3x + 4y + 0z = 0

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1)

Hence, the given line is perpendicular to z-axis.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** 8

Explanation:

We know that, dot product of two orthogonal vectors is always 0.

Hence,

$(2\hat{i} - \hat{j} + 2\hat{k})$. $(3\hat{i} + \lambda\hat{j} + \hat{k})$ = 0

$\Rightarrow$ 6 - λ + 2 = 0

$\Rightarrow$ λ = 8

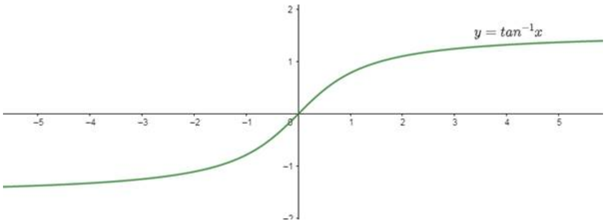
Section B

21. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - [\pi - \sec^{-1}2]$
$= \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{2}\right)$
$= -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}$

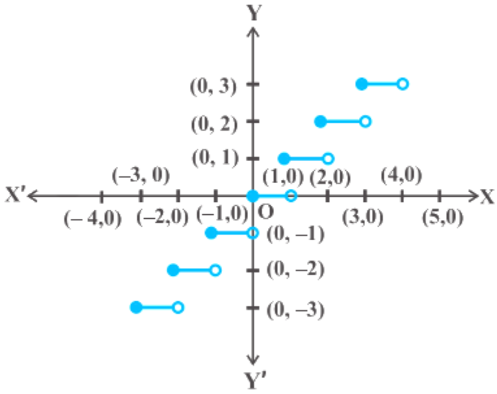
OR

Principal value branch of $\tan^{-1}$ x is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and its graph is shown below.



22. First, observe that f is defined for all real numbers. The graph of the function is given in the figure. From the graph, it looks like that f is discontinuous at every integral point. Below we explore if this is true.



Case 1: Let c be a real number which is not equal to any integer. It is evident from the graph that for all real numbers close to c the value of the function is equal to [c]; i.e., $\lim_{x \to c} f(x) = \lim_{x \to c} [x] = [c]$. Also f(c) = [c] and hence the function is continuous at all real numbers not equal to integers.

Case 2: Let c be an integer. Then we can find a sufficiently small real number r > 0 such that [c – r] = c – 1 whereas [c + r] = c. This, in terms of limits mean that

$\lim_{x \to c^-} f(x) = c - 1, \lim_{x \to c^+} f(x) = c$

Since these limits cannot be equal to each other for any c, the function is discontinuous at every integral point.

23. Given: f(x) = (x + 2)e^{-x}

f'(x) = e^{-x} – e^{-x} (x+2)

= e^{-x} (1 – x – 2)

= – e^{-x} (x+1)

For Critical points

f'(x) = 0

⇒ – e^{-x} (x + 1) = 0

⇒ x = – 1

Clearly $\mathrm{f}'(\mathrm{x}) > 0$ if $\mathrm{x} < -1$

$f'(x) < 0$ if $x > -1$

Hence f(x) increases in (–∞,–1), decreases in (–1, ∞)

OR

Taking log of both sides, we get

$30 \log x + 20 \log y = 50 \log(x + y)$

Differentiating both sides w.r.t. x, we get

$\frac{30}{x} + \frac{20}{y}\frac{dy}{dx} = \frac{50}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(\frac{20x-30y}{y(x+y)}\right) = \frac{20x-30y}{x(x+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

24. Let I = ∫ e^x (cot x - cosec^2 x) dx

Here, f(x) = cot x put e^x f(x) = t

f' (x) = -cosec^2 x

let e^x cot x = t

Diff. both sides w.r.t x

e^x cot x + e^x (-cosec^2x) = $\frac{dt}{dx}$

⇒e^x (cot x - cosec^2x) = dt

∴ ∫ e^x (cot x - cosec^2 dx) dx

= t + C [e^x cot x = t]

= e^x cot x + C

25. Given:

$A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$

To find the matrix as a result of the product AA^T

Firstly, we find the A^T(which is the transpose of the matrix A)

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

$\therefore AA^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ (Using the rule of matrix multiplication we get)

$= \begin{bmatrix} \cos x \times \cos x + (-\sin x) \times (-\sin x) & \sin x \times \cos x + (-\sin x) \times \cos x \\ \sin x \times \cos x + \cos x \times (-\sin x) & \sin x \times \sin x + \cos x \times \cos x \end{bmatrix}$

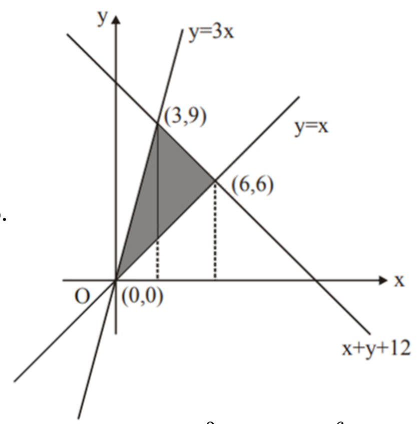
$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin x \cos x - \sin x \cos x \\ \sin x \cos x - \sin x \cos x & \sin^2 x + \cos^2 x \end{bmatrix}$

$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [$\because \cos^2 x + \sin^2 x = 1$ using the property of the trigonometry identity.]

$\Rightarrow AA^T = I_2$

Section C

26.



Required area = $\int_0^3 3x dx + \int_3^6 (12 - x) dx - \int_0^6 x dx$

$= 3\left[\frac{x^2}{2}\right]_0^3 + \left[12x - \frac{x^2}{2}\right]_3^6 - \left[\frac{x^2}{2}\right]_0^6$

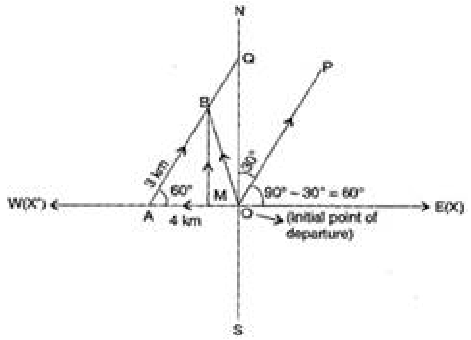
$= \frac{27}{2} + \frac{45}{2} - 18 = 18$ sq units

27. Let the initial point of departure is origin (0, 0) and the girl walks a distance OA = 4 km towards west.

Through the point A, draw a line AQ parallel to a line OP, which is 30^0 East of North, i.e., in East-North quadrant making an angle of 30^0 with North.

Again, let the girl walks a distance AB = 3 km along this direction $\overrightarrow{OQ}$

$\therefore \overrightarrow{OA} = 4\left(-\vec{i}\right) = -4\hat{i}$ ...(i) [$\because$ Vector $\overrightarrow{OA}$ is along OX']



Now, draw BM perpendicular to x - axis.

In ΔAMB by Triangle Law of Addition of vectors,

$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB} = (AM)\hat{i} + (MB)\hat{i}$

Dividing and multiplying by AB in R.H.S.,

$\overrightarrow{AB} = AB\frac{AM}{AB}\hat{i} + AB\frac{MB}{AB}\hat{j} = 3\cos 60^o\hat{i} + 3\sin 60^o\hat{j}$

$\Rightarrow AB = 3\frac{1}{2}\hat{i} + 3\frac{\sqrt{3}}{2}\hat{i} = \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}j$ ...(ii)

$\therefore$ Girl's displacement from her initial point O of departure to final point B,

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = -4\hat{i} + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}\right) = \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$\Rightarrow \overrightarrow{OB} = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

28. Let

$I = \int_0^x \frac{1}{1+\sin x}dx$

Multiplying Numerator and Denominator of the integrand by (1-sin x), gives

$I = \int_0^x \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}dx$

$= \int_0^x \frac{(1-\sin x)}{(1^2-\sin^2 x)}dx$

$= \int_0^x \frac{1-\sin x}{(\cos^2 x)}dx$

$= \int_0^x \frac{1}{\cos^2 x}dx - \int_0^\pi \frac{\sin x}{\cos^2 x}dx$

$= \int_0^x \sec^2 x dx - \int_0^x \tan x \cdot \sec x dx$

$I = [\tan x]_0^x - [\sec x]_0^x$

$= [\tan\pi - \tan 0] - [\sec\pi - \sec 0]$

= [0 - 0] - [-1 - 1]

= 2

$\therefore \int_0^x \frac{1}{1+\sin x}dx = 2$

OR

Let the given integral be,

I = $\int \frac{1}{5-4\cos x}dx$

Putting cos x = $\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\Rightarrow I = \int \frac{1}{5-4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$

$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{5\left(1+\tan^2\frac{x}{2}\right)-4+4\tan^2\frac{x}{2}}dx$

$= \int \frac{\sec^2\left(\frac{x}{2}\right)}{9\tan^2\frac{x}{2}+1}dx$

Let tan $\left(\frac{x}{2}\right)$ = t

$\Rightarrow \frac{1}{2}$ sec^2 $\left(\frac{x}{2}\right)$ dx = dt

sec^2 $\left(\frac{x}{2}\right)$ dx = 2dt

$\therefore I = \int \frac{2dt}{1-t^2-2t}$

$= 2\int \frac{-2dt}{t^2+2t-1}$

$= 2\int \frac{-2dt}{t^2+2t+1-2}$

$= \frac{2}{9} \times 3\tan^{-1}\left(\frac{t}{\frac{1}{3}}\right) + C$

$= \frac{2}{3}\tan^{-1}(3t) + C$

$= \frac{2}{3}\tan^{-1}\left(3\tan\frac{x}{2}\right) + C$

29. The given differential equation is,

$(x^2 + 1)\frac{dy}{dx} - 2xy = (x^2 + 1)(x^2 + 2)$

$\Rightarrow \frac{dy}{dx} + \left(\frac{-2x}{x^2+1}\right)y = x^2 + 2$

This is of the form $\frac{dy}{dx} + Py = Q$, where

$P = \frac{-2x}{x^2+1}$ and Q = x^2 + 2

Thus the given differential equation is linear differential equation

Now, $IF = e^{\int P\,dx}$

$= e^{\int \frac{-2x}{x^2+1} dx}$

$= e^{-\log(x^2+1)} = \left(x^2+1\right)^{-1} = \frac{1}{x^2+1}$

Therefore the solution is given by

$(IF) \cdot y = \int (IF) Q + C$

$\Rightarrow \frac{1}{x^2+1} \cdot y = \int \frac{1}{x^2+1}\left(x^2+2\right) dx + C$

$\Rightarrow \frac{y}{x^2+1} = \int \frac{x^2+2}{x^2+1} dx + C$

$\Rightarrow \frac{y}{x^2+1} = \int \frac{x^2+1+1}{x^2+1} dx + C$

$\Rightarrow \frac{y}{x^2+1} = \int \left\{1 + \frac{1}{x^2+1}\right\} dx + C$

$\Rightarrow \frac{y}{x^2+1} = x + \tan^{-1} x + C$

$\Rightarrow$ y = $(x^2 + 1)(x + \tan^{-1}x + C)$

OR

The given differential equation is,

$\left[x\sqrt{x^2+y^2} - y^2\right]$ dx + xy dy = 0

$\frac{dy}{dx} = \frac{y^2 - x\sqrt{x^2+y^2}}{xy}$

This is a homogeneous differential equation

Putting y = vx and $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$, we get

$v + x\frac{dv}{dx} = \frac{v^2x^2 - x\sqrt{x^2+v^2x^2}}{vx^2}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2 - \sqrt{1+v^2}}{v}$

$\Rightarrow v + x\frac{dv}{dx} = v - \frac{\sqrt{1+v^2}}{v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-\sqrt{1+v^2}}{v}$

$\Rightarrow \frac{v}{\sqrt{1+v^2}} dv = -\frac{1}{x} dx$

Putting 1 + v^2 = t, we get

v dv = $\frac{dt}{2}$

$\therefore \frac{1}{2\sqrt{t}} dt = -\frac{1}{x} dx$

Integrating both sides, we get

$\int \frac{1}{2\sqrt{t}} dt = -\int \frac{1}{x} dx$

$\Rightarrow \sqrt{t}$ = -log |x| + log C ...(i)

Substituting the value of t in (i), we get

$\sqrt{1+v^2} = \log\left|\frac{C}{x}\right|$

Hence, $\sqrt{y^2+x^2} = x\log\left|\frac{C}{x}\right|$ is the required solution.

30. We need to maximize z = x + y

First, we will convert the given inequations into equations, we obtain the following equations:

-2x + y = 1, x = 2, x + y = 3, x = 0 and y = 0

The line -2x + y = 1 meets the coordinate axis at $A\left(\frac{-1}{2}, 0\right)$ and B(0, 1). Join these points to obtain the line -2x + y = 1.

Clearly, (0, 0) satisfies the inequation -2x + y $\leq$ 1. So, the region in xy-plane that contains the origin represents the solution set of the given equation.

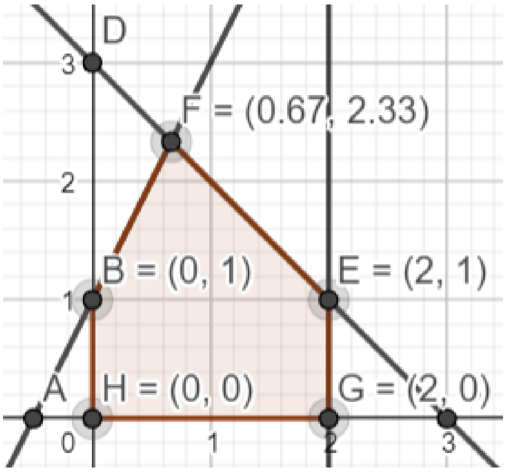
x = 2 is the line passing through (2,0) and parallel to the Y axis.

The region below the line x = 2 will satisfy the given inequation. The line x + y = 3 meets the coordinate axis at C(3, 0) and D(0, 3). Join these points to obtain the line x + y = 3.

Clearly, (0,0) satisfies the inequation x + y $\leq$ 3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

Region represented by x $\geq$ 0 and y $\geq$ 0 (non -negative restrictions)

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the regior represented by the inequations. These lines are drawn using a suitable scale.



The corner points of the feasible region are O(0,0), G(2,0), E(2,1) and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

The values of objective function at the corner points are as follows:

Corner point : Z = x + y

O(0, 0) : 0 + 0 = 0

C(2, 0) : 2 + 0=2

E(2, 1) : 2 + 1=3

$F\left(\frac{2}{3}, \frac{7}{3}\right) : \frac{2}{3} + \frac{7}{3} = \frac{9}{3} = 3$

We see that the maximum value of the objective function z is 3 which is at E(2,1) and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

Thus, the optimal value of objective function z is 3.

OR

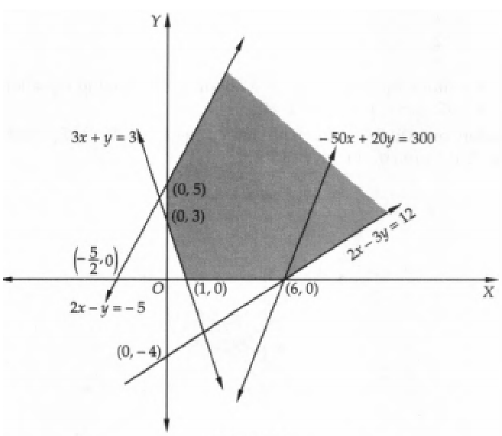
2x - y $\geq$ - 5

3x + y $\geq$ 3

2x - 3y $\leq$ 12

x $\geq$ 0, y $\geq$ 0

The feasible region of the system of inequations given in constraints is shown in a figure. We observe that the feasible region is unbounded.



The values of the objective function Z at the comer points are given in the following table:

Corner point (x, y)	Value of the objective function Z = -50x + 20y
(0,5)	Z = - 50 $\times$ 0 + 20 $\times$ 5 = 100
(0,3)	Z = - 50 $\times$ 0 + 20 $\times$ 3 = 60
(1,0)	Z = - 50 $\times$ 1 + 20 $\times$ 0 = - 50
(6,0)	Z = -50 $\times$ 6 + 20 $\times$ 0 = - 300

Clearly, - 300 is the smallest value of Z at the corner point (6, 0). Since the feasible region is unbounded, therefore, to check whether - 300 is the minimum value of Z, we draw the line - 300 = -50x + 20y and check whether the open half plane -50 x + 20y < -300 has points in common with the feasible region or not. From Fig., we find that the open half plane represented by - 50 x + 20y < - 300 has points in common with the feasible region. Therefore, Z = - 50x + 20y has no minimum value subject to the given constraints.

31. To show that the given function is continuous at x = 0, we show that

$(LHL)_{x=0} = (RHL)_{x=0} = f(0)$ ....(i)

Here, we have f(x) = $\begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$

Now, LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{4(1-\sqrt{1-x})}{x}$

$= \lim_{h\to 0} \frac{4[1-\sqrt{1-(0-h)}]}{0-h}$

$= \lim_{h\to 0} \frac{4[1-\sqrt{1+h}]}{-h}$

$= \lim_{h\to 0} \frac{4[1-\sqrt{1+h}]}{-h} \times \frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}$

$= \lim_{h\to 0} \frac{4\left[(1)^2-(\sqrt{1+h})^2\right]}{-h[1+\sqrt{1+h}]}$

$= \lim_{h\to 0} \frac{4[1-(1+h)]}{-h[1+\sqrt{1+h}]}$

$= \lim_{h\to 0} \frac{-h\times 4}{-h[1+\sqrt{1+h}]}$

$= \lim_{h\to 0} \frac{4}{1+\sqrt{1+h}}$

$= \frac{4}{1+\sqrt{1}} = \frac{4}{2}$ = 2

and RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \left(\frac{\sin x}{x} + \cos x\right)$

$\Rightarrow \quad \text{RHL} = \lim_{h\to 0} \left(\frac{\sin h}{h} + \cos h\right)$

$= \lim_{h\to 0} \frac{\sin h}{h} + \lim_{h\to 0} \cos h$

= 1 + cos 0

= 1 + 1

= 2

Also , given that x = 0, f(x) = 2 $\Rightarrow$f(0) = 2

Since, $(LHL)_{x=0} = (RHL)_{x=0} = f(0) = 2$

Therefore, f(x) is continuous at x = 0.

Section D

32. We have

$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(t+1)(t+2)}{(t+3)(t+4)}$, where x^2 = t

$= \frac{(t^2+3t+2)}{(t^2+7t+12)} = 1 - \frac{(4t+10)}{(t+3)(t+4)}$

Use partial fractions for 2nd part

Let $\frac{(4t+10)}{(t+3)(t+4)} = \frac{A}{(t+3)} + \frac{B}{(t+4)}$

$\Rightarrow$ (4t +10) = A(t + 4) + B(t + 3) .... (i)

Putting t = -3 in (i), we get A = -2

Putting t = -4 in (i), we get B = 6

$\therefore \frac{(4t+10)}{(t+3)(t+4)} = \frac{-2}{(t+3)} + \frac{6}{(t+4)}$ ..... (ii)

Thus, $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(t+1)(t+2)}{(t+3)(t+4)}$, where x^2 = t

$= \frac{(t^2+3t+2)}{(t^2+7t+12)} = 1 - \frac{(4t+10)}{(t+3)(t+4)}$

$= 1 - \left\{\frac{-2}{(t+3)} + \frac{6}{(t+4)}\right\}$ [from (ii)]

$= \left\{1 + \frac{2}{(t+3)} - \frac{6}{(t+4)}\right\}$

$= \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\}$

$\therefore \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = \int \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\} dx$

$= \int dx + 2\int \frac{dx}{(x^2+3)} - 6\int \frac{dx}{(x^2+4)}$

$= x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{6}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$

$= x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - 3 \tan^{-1}\left(\frac{x}{2}\right) + C$

33. Given that, A = R - {3}, B = R - {1}.

$f : A \to B$ is defined by $f(x) = \frac{x-2}{x-3}\ \forall x \in A$

For injectivity

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, f(x) is an injective function

For surjectivity

Let $y = \frac{x-2}{x-3} \Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = 2 - 3y $\Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow x = \frac{3y-2}{y-1} \in A,\ \forall y \in B$ [codomain]

So, f(x) is surjective function.

Hence, f(x) is a bijective function.

OR

Given that A = {1, 2, 3, ...9} (a, b) R (c, d) a + d = b + c for (a, b) ∈ A×A and (c, d) ∈ A×A.

Let (a, b) R (a, b)

⇒ a + b = b + a, ∀a, b ∈ A

Which is true for any a, b ∈ A

Hence, R is reflexive.

Let (a, b) R (c, d)

a+d = b+c

c + b = d + a ⇒ (c, d) R (a, b)

So, R is symmetric.

Let (a, b) R (c, d) and (c, d) R (e, f)

a + d = b + c and c + f = d + e

a + d = b + c and d + e = c + f(a + d) - (d + e) = (b + c) - (c + f)

(a - e) = b - f

a + f = b + e

(a, b) R (e, f)

So, R is transitive.

Hence R is an equivalence relation.

Let (a,b) R (2,5),then

a+5=b+2

a=b-3

If b<3 ,then a does not belong to A.

Therefore,possible values of b are >3.

For b=4,5,6,7,8,9

a=1,2,3,4,5,6

Therefore, equivalence class of (2,5) is

{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9).

34. We have, $A = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$ ...(i)

$\therefore |A| = 1(-3) - 2(-2) + 0 = 1 \neq 0$

Now, $A_{11} = -3, A_{12} = 2, A_{13} = 2,\ A_{21} = -2, A_{22} = 1, A_{23} = 1,\ A_{31} = -4, A_{32} = 2$ and $A_{33} = 3$

$\therefore adj\,(A) = \begin{vmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{vmatrix}^T = \begin{vmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{vmatrix}$

$\therefore A^{-1} = \frac{adjA}{|A|}$

$= \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ ...(i)

Also, we have the system of linear equations as

$x - 2y = 10$

$2x - y - z = 8$

and $-2y + z = 7$

Now, the given system of equations can be rewritten in the form AX=B,

where, $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Since A is non singular , therefore given system of equations has a unique solution given by,

$\therefore X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$= \begin{bmatrix} -30 + 60 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

$\therefore x = 0, y = -5$ and $z = -3$

35. Given Cartesian equations of lines

$\mathrm{L}_1 = \frac{\mathrm{x}-1}{3} = \frac{\mathrm{y}+1}{2} = \frac{\mathrm{z}-1}{5}$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Thus, vector equation of line L1 is

$\vec{\mathrm{r}} = (\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 5\hat{\mathrm{k}})$

And

$\mathrm{L}_2 : \frac{\mathrm{x}-2}{2} = \frac{\mathrm{y}-1}{3} = \frac{\mathrm{z}+1}{-2}$

Line L_2 is passing through point (2, -1, 1) and has direction ratios (2, 3, -2)

Thus, vector equation of line L2 is

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Now, to calculate distance between the lines,

$\vec{\mathrm{r}} = (\hat{\imath} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 5\hat{\mathrm{k}})$

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Here,we have

$\overrightarrow{a_1} = 1 - j + \hat{k}$

$\overrightarrow{\mathrm{b}_1} = 3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 5\hat{\mathrm{k}}$

$\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\overrightarrow{\mathrm{b}_2} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 2\hat{\mathrm{k}}$

Thus,

$\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{\mathrm{k}} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$

$= \hat{\imath}(-4 - 15) - \hat{\jmath}(-6 - 10) + \hat{k}(9 - 4)$

$\Rightarrow \overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = -19\hat{\mathrm{i}} + 16\hat{\mathrm{j}} + 5\hat{\mathrm{k}}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-19)^2 + 16^2 + 5^2}$
$= \sqrt{361 + 256 + 25}$
$= \sqrt{642}$
$\overrightarrow{a_2} - \overrightarrow{a_1} = (2-1)\hat{i} + (1+1)\hat{j} + (-1-1)\hat{k}$
$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 1 + 2\hat{j} - 2\hat{k}$
Now,
$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-19\hat{i} + 16\hat{j} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$
$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$
= -19 + 32 - 10
= 3
Thus, the shortest distance between the given lines is
$d = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right|$
$\Rightarrow d = \left|\frac{3}{\sqrt{642}}\right|$
$\therefore d = \frac{3}{\sqrt{642}}$ units
As $d \neq 0$
Hence, given lines do not intersect each other.

OR

Here,it is given that the equation of lines
$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$
$L2 = \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$
Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.

Suppose general point on line L_1 is P = (x_1, y_1, z_1)

$x_1 = 3s + 3$, $y_1 = -s + 8$, $z_1 = s + 3$

and suppose general point on line L_2 is Q = (x_2, y_2, z_2)

$x_2 = -3t - 3$, $y_2 = 2t - 7$, $Z_2 = 4t + 6$

$\therefore \overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)j + (4t + 6 - s - 3)\hat{k}$
$\therefore \overrightarrow{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)j + (4t - s + 3)\hat{k}$
Direction ratios of $\overrightarrow{PQ}$ are ((-3t - 3s - 6, (2t + s - 15), (4t - s + 3))
PQ will be the shortest distance if it perpendicular to both the given lines
Thus, by the condition of perpendicularity,
$\implies$ 3(-3t - 3s - 6) - 1(2t + s - 15) + 1(4t - s + 3) = 0 and
$\implies$ -3(-3t - 3s - 6) + 2(2t + s - 15) + 4(4t - s + 3) = 0
⇒ -9t - 9s - 18 -2t - s + 15 + 4t - s + 3 = 0 and
9t + 9s + 18 + 4t + 2s - 30 + 16t - 4s + 12 = 0
⇒ -7t - 11s = 0 and
29t + 7s = 0
Solving above two equations, we get,
t = 0 and s = 0
therefore,
P = (3, 8, 3) and Q = (-3, -7, 6)
Now distance between points P and Q is
$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$
$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$
$= \sqrt{36 + 225 + 9}$
$= \sqrt{270}$
$= 3\sqrt{30}$
Thus, the shortest distance between two given lines is

$d = 3\sqrt{30}$ units

Now, equation of line passing through points P and Q is,

$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$

$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$

$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$

$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

Thus, equation of line of shortest distance between two given lines is

$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

Section E

36. i. P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(Both are selected) = $P(A \cap B) = P(A) \cdot P(B)$ = $\frac{1}{3} \cdot \frac{1}{2}$

P(Both are selected) = $\frac{1}{6}$

ii. P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A' \cap B') = P(A') \cdot P(B')$ = $\frac{2}{3} \cdot \frac{1}{2}$

P(Both are selected) = $\frac{1}{3}$

iii. P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A') \cdot P(B) + P(A) \cdot P(B')$ = $\frac{2}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}$

P(Both are selected) = $\frac{3}{6} = \frac{1}{2}$

OR

P(A) = $\frac{1}{3}$, P(A') = $1 - \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = $1 - \frac{1}{3} = \frac{1}{2}$

P(atleast one of them selected) = 1 - P(none selected) = $1 - \frac{1}{3}$

P(atleast one of them selected) = $\frac{2}{3}$

37. i. If ₹ 15000 is invested in bond X, then the amount invested in bond Y = ₹ (35000 - 15000) = ₹ 20000.

A = Investment $\begin{bmatrix} X & Y \\ 15000 & 20000 \end{bmatrix}$

and B = $\begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \textit{Interest rate} \\ \begin{bmatrix} 10\% \\ 8\% \end{bmatrix} \end{array} = \begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \textit{Interest rate} \\ \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} \end{array}$

ii. The amount of interest received on each bond is given by

AB = $\begin{bmatrix} 15000 & 20000 \end{bmatrix} \times \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$

= [15000 $\times$ 0.1 + 20000 $\times$ 0.08] = [1500 + 1600] = 3100

iii. Let ₹ x be invested in bond X and then ₹ (35000 - x) will be invested in bond Y.

Now, total amount of interest is given by

$\begin{bmatrix} x & 35000 - x \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$ = [0.1x + (35000 - x)0.08]

But, it is given that total amount of interest = ₹ 3200

$\therefore$ 0.1x + 2800 - 0.08x = 3200

$\Rightarrow$ 0.02x = 400 $\Rightarrow$ x = 20000

Thus, ₹ 20000 invested in bond X and ₹ 35000 - ₹ 20000 = ₹ 15000 invested in bond Y.

OR

Let ₹ x invested in bond X, then we have

$x \times \frac{10}{100}$ = 500 $\Rightarrow$ x = 5000

Thus, amount invested in bond X is ₹ 5000 and so investment in bond Y be ₹ (35000 - 5000) = ₹ 30000

38. i. Since 'C' is cost of making tank

$\therefore$ C = 70xy + 45 $\times$ 2(2x + 2y)

$\Rightarrow$ C = 70xy + 90(2x + 2y)

$\Rightarrow$ C = 70xy + 180(x + y) [$\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}$]

$\Rightarrow$ C = 70x $\times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$

$\Rightarrow$ C = 280 + 180$\left(x + \frac{4}{x}\right)$

ii. x·y = 4

Volume of tank = length $\times$ breadth $\times$ height (Depth)

8 = $x \cdot y \cdot$ 2

$\Rightarrow$ 2xy = 8 $\Rightarrow$ xy = 4

iii. For maximum or minimum

$\frac{dC}{dx}$ = 0

$\frac{d}{dx}$(280 + 180(x + $\frac{4}{x}$)) = 0 $\Rightarrow$ 180$\left(1 + 4\left(-\frac{1}{x^2}\right)\right)$ = 0

$\Rightarrow$ 180$\left(1 - \frac{4}{x^2}\right)$ = 0 $\Rightarrow$ 1 $-$ $\frac{4}{x^2}$ = 0

$\Rightarrow \frac{4}{x^2}$ = 1 $\Rightarrow$ x^2 = 4

$\Rightarrow$ x = $\pm$2

$\Rightarrow$ x = 2 (length can never be negative)

OR

Now, $\frac{d^2C}{dx^2}$ = 180$\left(+\frac{8}{x^3}\right)$

$\Rightarrow \left.\frac{d^2C}{dx^2}\right|_{x=2}$ = 180 $\times \frac{8}{8}$ = 180 = +ve

Hence, to minimize C, x = 2m